

Money Management and Option Strategies

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Optimistic illusion is the handmaiden of irresponsible greed.

ABSTRACT

We describe a technique for computing the amount to wager given the expected return on an option strategy and the size of the account. Computations of the probability of profit, the maximum profit and maximum loss enable the computation of the expected return, however, this must be coupled with optimized money management. We add to this computation the Kelly criterion in order to compute the amount of the investment in the option activity. This enable a reduction in the risk of ruin, over many trials.

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I. INTRODUCTION

Given the dollar size of an account, we show how to compute the amount that should be wagered on a defined-risk option strategy. A defined-risk option or strategy has a finite amount that may be won or lost with a known probability of a win a loss.

Limits to our computations include the case when the risk is unbound. Examples of option strategies that have defined risk include iron condors, iron butterflies, etc. Examples of option strategies that we do not consider include naked call writing, straddle writing, etc. The unbound risk has the potential of infinite loss (in theory) and thus creates an ill-conditioned computation in our system.

II. THE KELLY CRITERION

The Kelly Criterion is used to obtain an optimal trading amount given the expected return on investment and the size of the account. Consider, for example, the coin-flip game where the win results in the payment of \$1 and the loss results in the loss of \$1. Starting with \$20 and a probability of a win set to 65% (and the probability of a loss set to 35%) what should the bet size be in order to optimize the return? The Kelly formula tells us that:

$$f = \frac{bp-q}{b}$$

Where:

f = fraction to bet based on funds.

$b = w/l$

p = the probability of a win.

q = the probability of a loss.

w = the amount you can win.

l = the amount you can lose.

This gives us the equation $f = \frac{1(0.65)-0.35}{1}$, which results in f being 0.3. This 0.3 indicates that a person should bet 30% of their funds ($0.3 * \$20 = \6) in order to bet an optimal amount. The bet amount is a function of the remaining portfolio. Let $r =$ the rate of growth of the portfolio so that

$$r = (1 + fw)^p(1 - fl)^q$$

Using the above parameters, we obtain a growth rate of:

$$r = (1 + .3)^{.65}(1 - .3)^{.35} = 1.04676$$

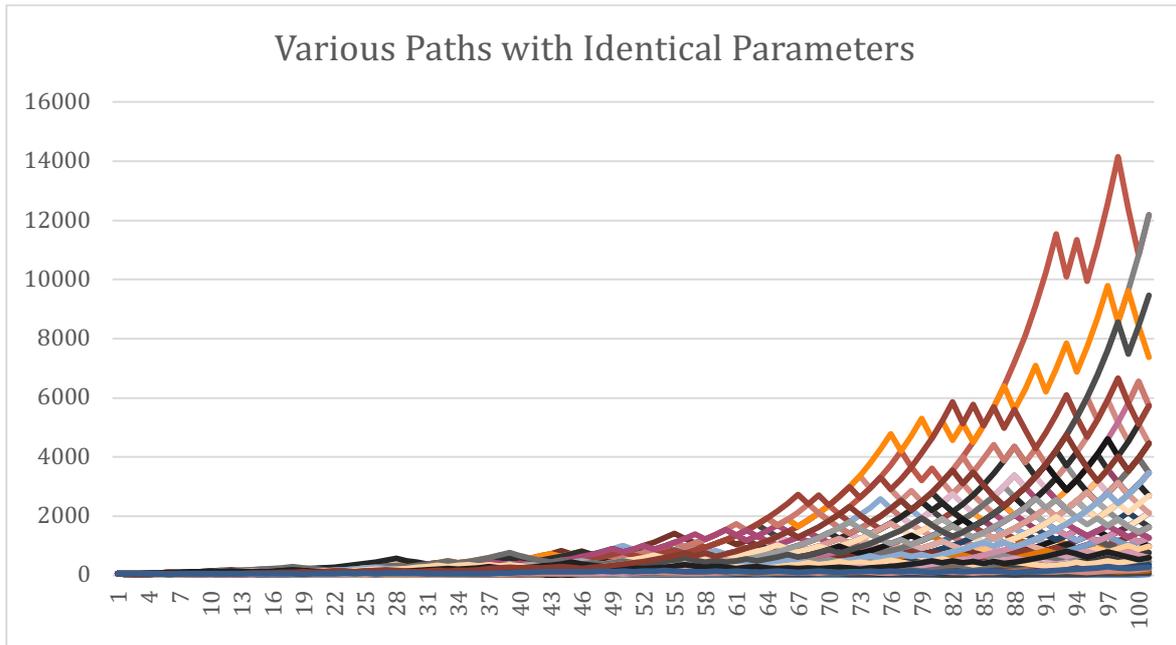


Figure 1a. Variation in Outcome (Longer run)

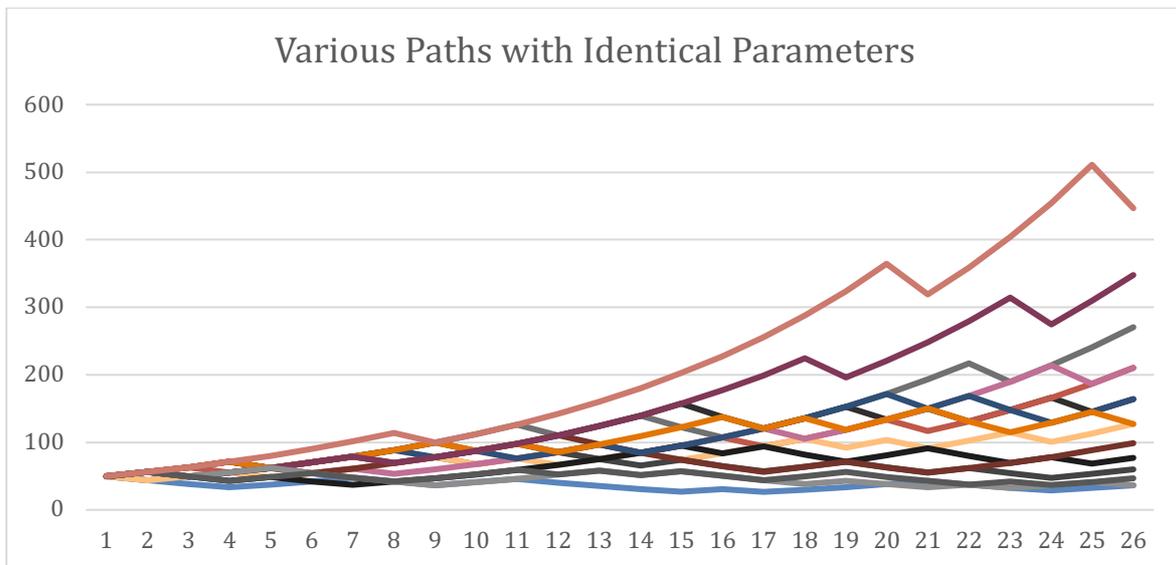


Figure 1b. Variation in Outcome (Shorter run)

Figures 1a and 1b show the variation in outcomes for several experiments with identical parameters. The Kelly fraction gives an optimal value, but as seen in the graph above some of the sample trials proved exceedingly lucky as others shows themselves to be less-effective, but never reaching 0 given the characteristics associated with using Kelly.

Suppose a person's starting equity is \$7500 with a win probability of 60% and an expected return of 4:1. This is where the Kelly Criterion steps in and gives the following based on the current situation:

Given:

- A starting balance of \$7500
- $b = 4:1$ (Expected return of stock) (\$4 for every dollar risked)
- $p = 60\%$ (Win probability)
- $q = 40\%$ (Lose probability)

In this situation we get $f = \frac{4(0.6)-0.4}{4}$, which results in f being 0.5 of \$7,500 = \$3,750 in order to maximize return.

Code for simulation

```

import java.lang.Math;
import java.io.FileWriter;
import java.io.IOException;

public class Kelly {
//Setting starting balance
Double bal = 50;
Double startingBal = bal;

//Player number and attempt per each one
Int numberOfPlayers = 100;
Int numberOfAttempts = 100;

//Winning probabilities
double p = 0.65;
double q = 1 - p;

//Ratio of win to lose
double w = 4;
double l = 6;
double b = w/l;
FileWriter myWriter;

Public Kelly() throws IOException {
myWriter = new FileWriter("filename.csv");
}

Private void gameTrialRuns(int playerNum) throws IOException {
for(int i = 0; i < numberOfPlayers; i++){
System.out.println("Player " + playerNum);
System.out.println("Your starting bal is " + bal);
myWriter.write(bal + ",");
for(int j = 0; j < numberOfAttempts; j++){
double kellyBetAmount = getKellyBetAmount();
double roundedKelly = ((double) Math.round(kellyBetAmount * 100) / 100);
System.out.println("New betting amount based on balance: " + roundedKelly);
double x = Math.random();
if(x >= p){
bal = getKellyLossCase(myWriter,
bal, kellyBetAmount);
if(bal <= 0){
System.out.println("You lost");
break;
}
}
else{
bal = getKellyWinCase(myWriter,
bal, kellyBetAmount);
}
}
myWriter.append("\n");
System.out.println("Ending bal: " + bal);
playerNum++;
bal = startingBal;
System.out.println();
}
}
}

```

```

private double getKellyWinCase(FileWritermyWriter, double bal, double
kellyBetAmount) throws IOException {
System.out.println("You won this bet");
bal = bal + kellyBetAmount;
bal = ((double) Math.round(bal * 100) / 100);
myWriter.write(bal + ",");
System.out.println("Your new bal is " + bal);
return bal;
}

private doublegetKellyLossCase(FileWritermyWriter, double bal, double
kellyBetAmount) throws IOException {
System.out.println("You lost this bet");
bal = bal - kellyBetAmount;
bal = ((double) Math.round(bal * 100) / 100);
System.out.println("Your new bal is " + bal);
myWriter.write(bal + ",");
return bal;
}

public void run() {

try {
//Calculating kelly value

int playerNum = 1;

//Runs player simulation using kelly theory
gameTrialRuns(playerNum);
myWriter.close();
System.out.println("Successfully wrote to the file.");
} catch (IOException e) {
System.out.println("An error occurred.");
e.printStackTrace();
}

}

private double getKellyBetAmount() {
double f = ((b * p) - q) / b);
return bal * f;
}

public static void main(String[] args) {
try {
Kelly k = new Kelly();
k.run();
} catch (IOException e) {
e.printStackTrace();
}

}
}

```

The Java code simulates a coin flip that makes use of the Kelly criterion. Along with it are snippets of a sample simulation that shows an output from the code of 4 runs (of 10 attempts each) computing the average ending balance across all players. As seen in the code the odds for a win in this are 65 percent but the win amount is \$1 and the loss amount is \$1. This code generates data that can then be used to better visualize example results when utilizing concepts from the Kelly criterion.

Player 1	Player 2	Player 3	Player 4
Your starting bal is 50.0	Your starting bal is 50.0	Your starting bal is 50.0	Your starting bal is 50.0
New betting amount based on balance: 6.25	New betting amount based on balance: 6.25	New betting amount based on balance: 6.25	New betting amount based on balance: 6.25
You lost this bet	You won this bet	You won this bet	You lost this bet
Your new bal is 43.75	Your new bal is 56.25	Your new bal is 56.25	Your new bal is 43.75
New betting amount based on balance: 5.47	New betting amount based on balance: 7.03	New betting amount based on balance: 7.03	New betting amount based on balance: 5.47
You won this bet	You won this bet	You won this bet	You won this bet
Your new bal is 49.22	Your new bal is 63.28	Your new bal is 63.28	Your new bal is 49.22
New betting amount based on balance: 6.15	New betting amount based on balance: 7.91	New betting amount based on balance: 7.91	New betting amount based on balance: 6.15
You won this bet	You lost this bet	You won this bet	You lost this bet
Your new bal is 55.37	Your new bal is 55.37	Your new bal is 71.19	Your new bal is 43.07
New betting amount based on balance: 6.92	New betting amount based on balance: 6.92	New betting amount based on balance: 8.9	New betting amount based on balance: 5.38
You lost this bet	You won this bet	You won this bet	You lost this bet
Your new bal is 48.45	Your new bal is 62.29	Your new bal is 80.09	Your new bal is 37.69
New betting amount based on balance: 6.06	New betting amount based on balance: 7.79	New betting amount based on balance: 10.01	New betting amount based on balance: 4.71
You won this bet	You won this bet	You won this bet	You lost this bet
Your new bal is 54.51	Your new bal is 70.08	Your new bal is 90.1	Your new bal is 32.98
New betting amount based on balance: 6.81	New betting amount based on balance: 8.76	New betting amount based on balance: 11.26	New betting amount based on balance: 4.12
You won this bet	You won this bet	You lost this bet	You won this bet
Your new bal is 61.32	Your new bal is 78.84	Your new bal is 78.84	Your new bal is 37.1
New betting amount based on balance: 7.67	New betting amount based on balance: 9.86	New betting amount based on balance: 9.86	New betting amount based on balance: 4.64
You lost this bet	You won this bet	You won this bet	You won this bet
Your new bal is 53.65	Your new bal is 88.7	Your new bal is 88.7	Your new bal is 41.74
New betting amount based on balance: 6.71	New betting amount based on balance: 11.09	New betting amount based on balance: 11.09	New betting amount based on balance: 5.22
You won this bet	You won this bet	You won this bet	You won this bet
Your new bal is 60.36	Your new bal is 99.79	Your new bal is 99.79	Your new bal is 46.96
New betting amount based on balance: 7.55	New betting amount based on balance: 12.47	New betting amount based on balance: 12.47	New betting amount based on balance: 5.87
You won this bet	You lost this bet	You won this bet	You won this bet
Your new bal is 67.91	Your new bal is 87.32	Your new bal is 112.26	Your new bal is 52.83
New betting amount based on balance: 8.49	New betting amount based on balance: 10.92	New betting amount based on balance: 14.03	New betting amount based on balance: 6.6
You won this bet	You won this bet	You won this bet	You lost this bet
Your new bal is 76.4	Your new bal is 98.24	Your new bal is 126.29	Your new bal is 46.23
Ending bal: 76.4	Ending bal: 98.24	Ending bal: 126.29	Ending bal: 46.23

1. COMPUTATION OF EXPECTED RETURN

Suppose you are given an ideal coin such that the probability of a head is equal to the probability of a tail and that all probabilities sum to one. Assume also that there are only two outcomes. We represent the two outcomes with:

$$f(k; p) = \begin{cases} p, & \text{if } k = 1 \\ 1 - p, & \text{if } k = 0 \end{cases} \quad (1)$$

Where $k=1$ corresponds to the outcome of flipping a coin resulting in a head, and $k=0$ for a tail. For an ideal coin, $p = 0.5$. Suppose that every time you get $k=1$, you get paid \$2 and if you get $k=0$ pay out \$1. The expected value of the coin flip must therefore be:

$$E(f(k; p)) = 2p - (1 - p) = \frac{2}{2} - \frac{1}{2} = 0.5 \quad (1)$$

That is, you must multiply the probability of a win times the amount you win and subtract from that the probability that you lose times the amount of the loss. This simple idea is expressed by:

$$E(f(k; p)) = M_p p - M_l (1 - p) = \frac{2}{2} - \frac{1}{2} = 0.5 \quad (2)$$

where

M_p = max profit
 M_l = max loss

2. KELLY AND THE JADE LIZARD

In the Jade Lizard option strategy, we write a call and a put while buying a call at a higher strike price. We select a company called Clarus Corporation (symbol CLAR). They are trading at \$8.25. Using TastyWorks (a platform that can report the probability of a profit) we set up our trade as shown in Figure 2.



Figure 2. A Jade Lizard for CLAR

We see that the POP (Probability of Profit) is 99.5% and that we are paid a credit of \$4.10 with 16d (16 days) until expiration. This credit represents our maximum profit, per contract (100 shares * \$4.10/share = \$410). Because one leg of the Jade lizard is a naked put at \$5, we can lose \$5 of intrinsic value in the put if the stock goes to zero in 16 days. However, because we have already been paid \$4.10 per share, our max loss is \$5-\$4.10=\$0.90. For 100 shares our maximum profit is therefore \$410, and our maximum loss is \$90. To compute our expected return, we write:

$$E(f(k; p)) = w_p p - \ell_r(q) = 410(.995) - 90(.005) = 407.5$$

Naturally, this scheme can scale. That is, we do not have to trade just one Jade Lizard, we could trade 10 or 100 Jade Lizards with expected returns of \$4,075 or \$40,750. However, with little volume this may not be a practical trade. In fact, we may not even be able to trade a single Jade Lizard. For the Kelly criterion, we compute the fraction for the bet using:

$$f = \frac{bp - q}{b} = \frac{(410/90).995 - .005}{(410/90)} = .99$$

The assertion that we have a binary payoff is without foundation. That is the payoff (or loss) will also have a probability associated with it and this is not reflected in the binomial model of Kelly.

3. SELECTING JADE LIZARD STOCKS

The astute reader may be wondering how to select the stock for the Jade Lizard. With a the naked put, as shown in Section 4, we are exposed to a stock that can go to zero. Thus, selecting a low-priced stock with a low-priced put, along with a call spread, we obtain a synthetic reverse Iron Condor in profit profile (that is we are selling the body and purchasing the wings. The sale of the body is a sale of a straddle (both the call and the put). Being naked on the put side is ok since the stock cannot go below zero (thus limiting our loss). However, from a margin perspective, the sale of the call without the corresponding purchase of high-strike call would expose us to unlimited loss. Unlimited loss will also cause our expected return computation to tend toward minus infinity.

How could this happen? Suppose you had a naked short call and during the next 16 days Clarus got a take-over bid for \$100 a share. At that point a naked call is assigned, and you are living in *Mudville*. As unlikely as that may be, you must therefore have the higher-priced call, even if the likely-hood of a take-over is trivial. Operating without such insurance is the moral equivalent of picking up quarters in front of a steam roller.

4. THE REVERSE IRON CONDOR

In the case of the higher priced stock, we select *Nabors* (symbol NBR). We structure the Reverse Iron Condor so that we buy the guts and sell the wings (for cost off-set). Figure 3 shows that the wings are positioned close to a single standard deviation away from a Gaussian distributed probability density function, computed using observed price action.



Figure 3. Sell the guts and buy the wings.

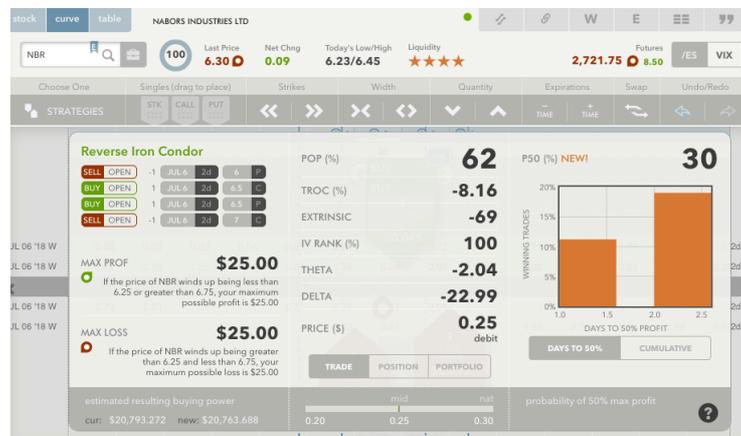


Figure 4. Shows the max profit and max loss.

While the maximum profit and loss are the same, \$25, the probability is skewed toward a probability of profit (62%) so that:

$$E(f(k; p)) = w_p p - \ell_l(q) = 25(.6) - 25(.4) = 6$$

The \$6 expected return is low, however, with the number of days to go equal to two, the ROI on a yearly basis is high (we can conduct such trades more often). The max loss is \$25, no matter how dire the exogenous elements appear. After all, it is hard to accurately predict war (or tweets).

5. THE IRON CONDOR

In the case of the Iron Condor, we sell the guts and purchase the wings. The wings are our insurance against a black swan event. In the case of higher-priced stocks, putting on strategies with naked legs will eventually trip-up even the most disciplined trader. This can result in a life changing event from which you may never recover (*Jesse Livermore* was worth \$100 million in 1929, after he lost all his money, he shot himself in the head). The statistical arbitrageur must always be mindful that there is a non-zero probability of losing money on any given trade. Large gambles on any single trade negate the advantage obtained from repeated trials.

For our Iron Condor, we select a stock with a high-IMPLIED volatility rank (a high IV), Tesla (symbol TSLA), as shown in Figure 5;



Figure 5. Sell the Gut and Buy the Wings

The Iron Condor provides a credit of \$3.31 with a 61% probability of profit. A maximum loss is \$1.69 (which occurs if the stock is less than \$296.69). This gives us an expected return of:

$$E(f(k; p)) = w_p p - \ell(q) = 331(.61) - 169(.39) = 136$$

\$136 per Iron Condor.

6. SELLING A CALL BUTTERFLY

In selling a call butterfly, we sell the wings with an in-the-money call and an out of the money call. We then purchase 2 of the at the money calls. For Alibaba (symbol BABA) we show the trade in Figure 6:



Figure 6. Sell a Call Butterfly

With one day to go, we have a 2.21 credit, our maximum profit. Our maximum loss occurs when the price winds up greater than 184.71 or less than 185.30, as shown in Figure 7:

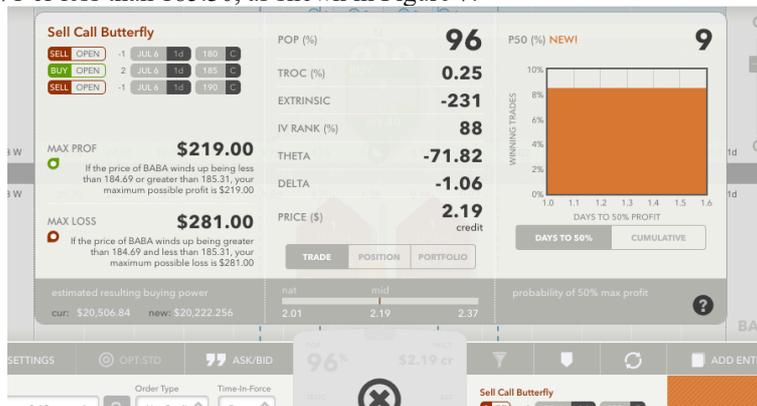


Figure 7. Defined Risk and a High Probability Butterfly

The probability of a profit is 96% and this more than makes up for the maximum loss being larger than the maximum profit. Tariffs on China will take effect on July 6th (it is July 5th now). Perhaps the Tariffs are already baked into the share price, but:

$$E(f(k;p)) = w_p p - \ell_f(q) = 219(.96) - 281(.04) = 199$$

Which indicates the market may not have baked the expected tariff enforcement into the stock price action.

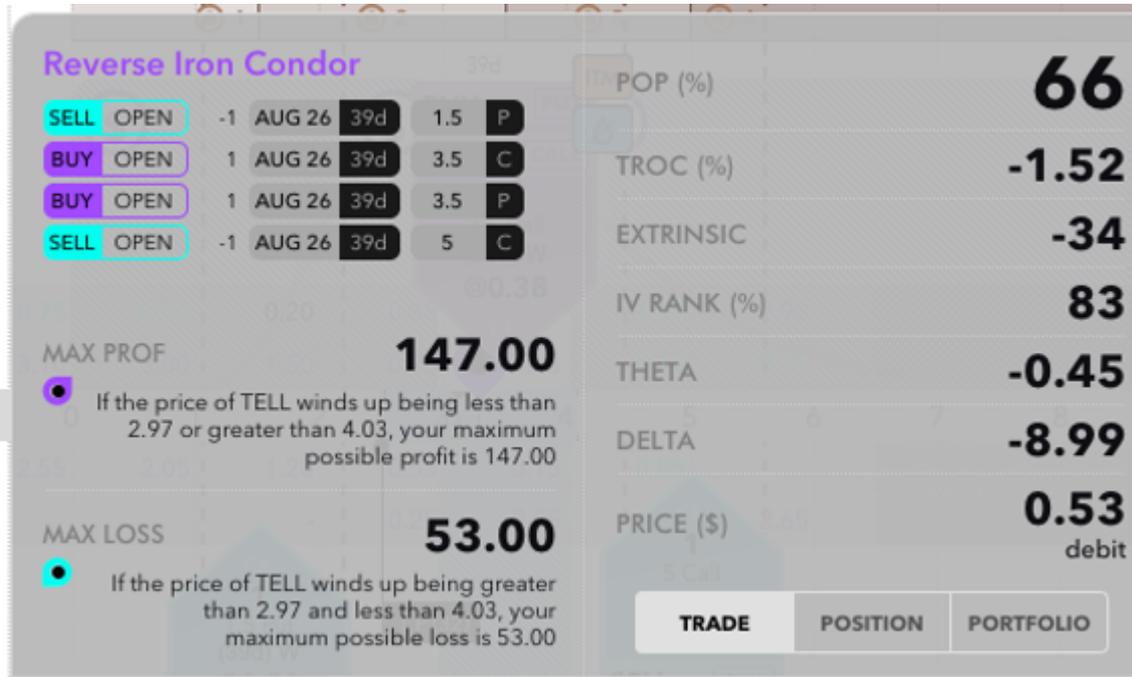


Figure 8. Defined Risk and a High Probability Butterfly with Kelly

Figure 8 shows a 39-day reverse iron condor with strike prices in the body the same (also known as an iron butterfly, or iron fly, for short). Kelly criterion gives.

$$f = \frac{bp-q}{b} = \frac{(147/53).66-.34}{(147/53)} = .53$$

Kelly criterion gives 0.53 and, if we use \$100 as the portfolio size, \$53 would be the max we would have to bet (and is our max loss).

7. PROOF OF KELLY FORMULA

$$B_n = (1 + fw)^w * (1 - f)^\ell B$$

Where B_n = Bet

$$Gain_n = (1 + fw)^w * (1 - f)^\ell$$

Where $Gain_n$ = Gain after trials

The geometric mean G, is obtained by taking the limit as $n \rightarrow \infty$ of the nth root of the gain

$$G = \lim_{n \rightarrow \infty} ((1 + fw)^{\frac{w}{n}} * (1 - f)^{\frac{\ell}{n}})^n$$

which is

$$G = (1 + fw)^p * (1 - f)^q$$

The value of your bankroll after n games is

Where G = Gain potential

$$B_n = G^n * B$$

To find the f that maximizes G , we take the derivative of $G = (1 + fw)^p * (1 - f)^q$

Let's plot G as a function of (f) holding W , P , and Q constant with respect to f and set it equal to 0:

$$(1 + fw)^p * (-q(1 - f)^{q-1}) + wp(1 + fw)^{p-1} * (1 - f)^q = 0$$

Solving for f gives

$$f = \frac{pw - q}{w} = \frac{p - q}{w}$$

With $p = 0.65$, $q = 0.35$, and $b = 1$ we obtain a growth rate of:

$$r = (1 + .3)^{.65} (1 - .3)^{.35} = 1.04676$$

and

$$f = \frac{1(0.65) - 0.35}{1} = 0.3$$

These formulas indicating that $f=0.3$ maximizes growth rate at 1.04676. Using excel this value can be further proven for validity.

f	Growth Rate r		
		26.00%	1.045856154
0.00%	1	27.00%	1.046249876
1.00%	1.002954453	28.00%	1.046532823
2.00%	1.005817617	29.00%	1.046703638
3.00%	1.008589176	30.00%	1.046760903
4.00%	1.011268785	31.00%	1.046703133
5.00%	1.013856071	32.00%	1.046528777
6.00%	1.016350631	33.00%	1.046236212
7.00%	1.018752031	34.00%	1.045823741
8.00%	1.021059806	35.00%	1.045289588
9.00%	1.023273462	36.00%	1.044631894
10.00%	1.02539247	37.00%	1.043848715
11.00%	1.027416269	38.00%	1.042938014
12.00%	1.029344267	39.00%	1.041897659
13.00%	1.031175834	40.00%	1.040725416
14.00%	1.032910307	41.00%	1.039418943
15.00%	1.034546987	42.00%	1.037975784
16.00%	1.036085136	43.00%	1.036393365
17.00%	1.037523978	44.00%	1.034668981
18.00%	1.038862699	45.00%	1.032799794
19.00%	1.040100443	46.00%	1.030782821
20.00%	1.041236312	47.00%	1.028614926
21.00%	1.042269364	48.00%	1.026292808
22.00%	1.043198613	49.00%	1.023812991
23.00%	1.044023024		
24.00%	1.044741515		
25.00%	1.045352954		

Figure 9 Percentage Value vs Growth Rate

In figure 9 when incrementing the f value to be used for calculating the growth rate values are given, but keep in mind finding that the maximum growth rate per percent is the desired end goal.

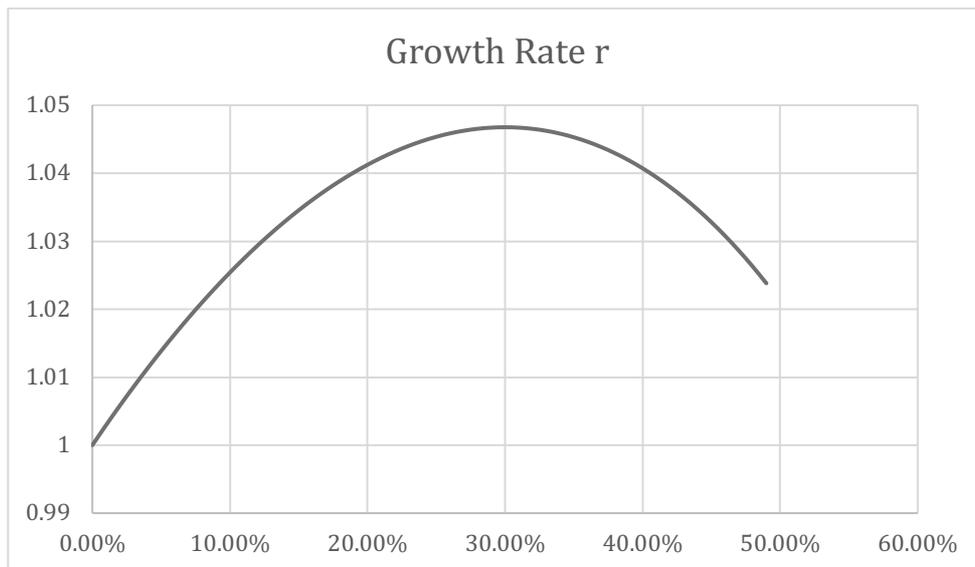


Figure 10. Percentage Value vs Growth Rate

Figure 10 shows a plot of the values from Figure 9. Figures 9 and 10 show the growth rate and percentage values align produce the maximum percentage.

The key is to apply this exact same logic for every new turn that a person plays rather than sticking to a singular bet value throughout the whole run which will has a non-zero probability of ruin.

8. SUMMARY

The amount to bet is just as important as what to bet on, and this can be computed for defined risk trades that with an expected return. The question of how accurate the probabilities are remains open.

One topic for future research is to determine how much error these probabilities have, as a function of time.

REFERENCES

- [1]. Christensen, Morten Mosegaard. "On the history of the growth optimal portfolio." Preprint, University of Southern Denmark 389 (2005).
- [2]. Kelly, John L, Jr., A New Interpretation of Information Rate, Bell Systems Technical Journal, Vol. 35, pp. 917- 926, 1956
- [3]. Louis M. Rotando, and Edward O. Thorp. "The Kelly Criterion and the Stock Market." The American Mathematical Monthly, vol. 99, no. 10, 1992, pp. 922-31. JSTOR, <https://doi.org/10.2307/2324484>. Accessed 5 Aug. 2022.
- [4]. Poundstone, William, "Fortunes Formula: The Untold Story of the Scientific Betting System that Beat the Casinos and Wall Street," Hill and Wang, New York, NY, 2005